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# Variational solution of the problem of unsteady-state convective heat transfer in the channel

N. M. TSIREL'MAN
The Sergo Ordzhonikidze Ufa Aviation Institute, 450000, Ufa, U.S.S.R.

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**Abstract**—With the use of a convolution-type functional, the variational description is given for the process of unsteady-state convective heat transfer in channels of complex cross-section with an arbitrary dependence of the velocity vector, thermophysical characteristics of the medium, volumetric heat generation source and of the parameters of boundary conditions on space and time. Based on the Galerkin–Kantorovich method involving the reduction to ordinary differential equations, a corresponding system of Euler equations is written out the solution of which (analytical or numerical) is necessary to determine the temperature field in each specific case. An example is provided to illustrate a numerical solution of the problem stated.

## 1. INTRODUCTION

It is well known that the greatest progress has been made in the determination of the steady-state thermal state of inner laminar streams in the hydrodynamically steady state and stabilized flow with constant thermophysical characteristics of the medium [1–3]. For the laminar or turbulent regimes of such a flow, the allowance for the dependence of the thermophysical characteristics of the medium, volumetric heat generation source and of the parameters of boundary conditions on coordinates on the basis of the variational description of the phenomenon studied was made in ref. [4]. The methods for predicting unsteady-state temperature fields in hydrodynamically steady state and stabilized flows with a variety of simplifying assumptions were the concern of refs. [2, 5]. On the other hand, for practical purposes, it is especially important to know convective heat exchange in such cases when a laminar or turbulent flow in a channel of arbitrary cross-section can be unsteady-state and hydrodynamically nonstabilized, whereas the thermophysical characteristics of the medium, volumetric heat generation sources and the parameters of the conditions for the solution uniqueness depend arbitrarily on the problem arguments, i.e. coordinates and time.

## 2. STATEMENT OF THE PROBLEM AND CONSTRUCTION OF ITS VARIATIONAL DESCRIPTION

The internal problem of convective heat exchange, which, due to its complexity, has not been posed earlier for obtaining an approximate analytical solution or for simplifying computer calculations because of the necessity of considering the hydrodynamic, in addition to the thermal, unsteady state, non-stabilized characteristics of the medium, boundary conditions and volumetric heat generation sources on coordinates and time, has the following form with respect to the unknown temperature $T(x,y,z,\tau)$:

$$CT_\tau + Cw_1T_x + Cw_2T_y + Cw_3T_z = \frac{\partial}{\partial y}(\lambda T_y) + \frac{\partial}{\partial z}(\lambda T_z) + q_v, \quad x>0, (y,z)\in\Omega, \tau>0 \quad (1)$$

under the initial condition

$$T(x,y,z,0) = T_0(x,y,z), \quad x>0, (y,z)\in\Omega \quad (2)$$

with the condition in the inlet section

$$T(0,y,z,\tau) = T_{en}(y,z,\tau), \quad (y,z)\in\Omega, \tau>0 \quad (3)$$

and with the boundary conditions of the first, second or third kind on the parts of the channel bounding surface

$$T(x,y,z,\tau) = T_w(x,y,z,\tau), \quad x>0, (y,z)\in\Gamma_1, \tau>0 \quad (4a)$$

$$-\lambda(x,y,z,\tau)\nabla T\cdot \mathbf{n} = q(x,y,z,\tau), \quad x>0, (y,z)\in\Gamma_2, \tau>0 \quad (4b)$$

$$-\lambda(x,y,z,\tau)\nabla T\cdot \mathbf{n} = \alpha(x,y,z,\tau)[T(x,y,z,\tau) - T_f(x,y,z,\tau)], \quad x>0, (y,z)\in\Gamma_3, \tau>0. \quad (4c)$$

Figure 1 shows the geometric region where heat transfer takes place in the channel.

Now, a variational description of the problem formulated will be constructed with the use of the functional

$$J(T) = \sum_{i=1}^{6} J_i(T) \quad (5)$$

in which

$$J_1(T) = \int_v \left[ RT_\tau + GT_x + BT + DT_y + ET_z + \frac{\partial}{\partial y}(AT_y) + \frac{\partial}{\partial z}(AT_z) + Q \right] T(X-x, t-\tau)\,\mathrm{d}v \quad (6)$$

## NOMENCLATURE

$C(x, y, z, \tau)$ volumetric heat capacity of flow
$q(x, y, z, \tau)$ heat flux density over $\Gamma_2(0, x)$
$q_v(x, y, z, \tau)$ power of volumetric heat generation sources
$T(x, y, z, \tau)$ temperature in a channel at the point with coordinates $x, y, z$ at time $\tau$
$T_{en}(y, z, \tau)$ temperature in entrance section of channel (at $x = 0$)
$T_f(x, y, z, \tau)$ outer medium temperature over $\Gamma_3(0, x)$
$T_w(x, y, z, \tau)$ surface temperature over $\Gamma_1(0, x)$
$T_0(x, y, z)$ initial temperature in a channel at the point with coordinates $x, y, z$ (at $\tau = 0$)
$w_1(x, y, z, \tau)$, $w_2(x, y, z, \tau)$, $w_3(x, y, z, \tau)$ velocity vector projections on axes $x, y, z$, respectively
$x$ direction of liquid (gas) flow.

Greek symbols
$\alpha(x, y, z, \tau)$ convective heat transfer coefficient
$\Gamma$ $\Gamma_1 \bigcup \Gamma_2 \bigcup \Gamma_3$, piecewise-smooth curve with parts $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ bounding channel cross-section
$\lambda(x, y, z, \tau)$ thermal conductivity coefficient
$\Omega(y, z)$ channel cross-section.

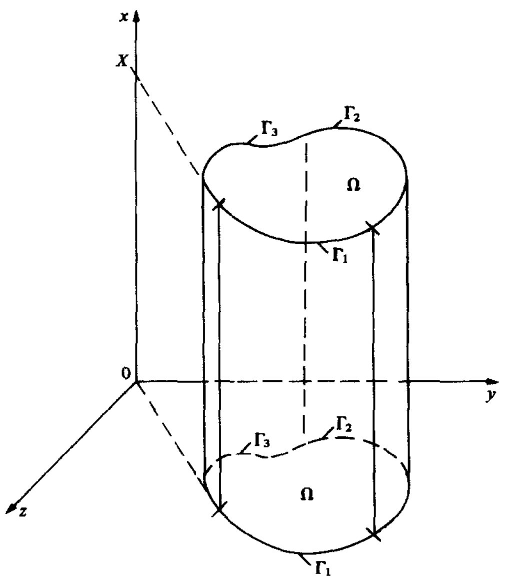

FIG. 1. Region where the phenomenon studied takes place.

$$J_2(T) = \int_{S_1} [aT(x, 0) + b]T(X - x, t)\,ds \quad (7)$$

$$J_3(T) = \int_{S_2} [\tilde{a}T(0, \tau) + \tilde{b}]T(X, t - \tau)\,ds \quad (8)$$

$$J_4(T) = \int_{S_3}\int_{\Gamma_1} \{p_1[T(\tau) - T_w]\nabla T^*\mathbf{n} + p_2 T(\tau)\nabla T^*\mathbf{n} + p_3 T \cdot T^*\}\,dl\,ds \quad (9)$$

$$J_5(T) = \int_{S_3}\int_{\Gamma_2} [a_1(\lambda\nabla T\mathbf{n} + q)T^* + a_2 T(\tau)\nabla T^*\mathbf{n} + a_3 T \cdot T^*]\,dl\,ds \quad (10)$$

$$J_6(T) = \int_{S_3}\int_{\Gamma_3} \{b_1[\lambda\nabla T\mathbf{n} + \alpha(T - T_f)]T^* + g_1 T^*\nabla T \cdot \mathbf{n} + g_2 T \cdot T^*\}\,dl\,ds. \quad (11)$$

In formulae (7)–(11) the arguments of $T$, $\nabla T$, $T_w$, $T_f$, $q$, $\alpha$, $\lambda$ and in the as yet unknown functions $A$, $B$, $G$, $E$, $D$, $Q$, $R$, $a$, $b$, $\tilde{a}$, $\tilde{b}$, $a_1$, $a_2$, $a_3$, $b_1$, $g_1$, $g_2$, $p_1$, $p_2$, $p_3$ are omitted except for the most indispensable ones. Besides, the integrals and $T^*$ in the above equations are thus

$$\int_v dv = \int_0^X \int_0^t \int_\Omega dy\,dz\,d\tau\,dx$$

$$\int_{S_1} ds = \int_0^X \int_\Omega dy\,dz\,dx$$

$$\int_{S_2} ds = \int_0^t \int_\Omega dy\,dz\,d\tau$$

$$\int_{S_3}\int_{\Gamma_i} dl\,ds = \int_0^X \int_0^t \int_{\Gamma_i} dy\,dz\,d\tau\,dx$$

$$T^* = T(X - x, y, z, t - \tau).$$

With the use of the results obtained in the Appendix, the first variation of functional (5) will have the form

$$\delta J(T) = \int_v \left[\frac{\partial}{\partial y}(\lambda T_y) + \frac{\partial}{\partial z}(\lambda T_z) - CT_\tau - Cw_1 T_x - Cw_2 T_y - Cw_3 T_z + q_v\right]\delta T^*\,dv$$

$$- \int_{S_1} C(x, y, z, 0)[T(x, 0) - T_0]\delta T(X - x, t)\,ds$$

$$-\int_{S_2} Cw_1(0,y,z,\tau)[T(0,\tau)-T_{en}]\delta T(X,t-\tau)\,ds$$
$$+\int_{S_3}\int_{\Gamma_1}\lambda(T-T_w)\delta\nabla T^*\mathbf{n}\,dl\,ds$$
$$-\int_{S_3}\int_{\Gamma_2}(\lambda\nabla T\cdot\mathbf{n}+q)\delta T^*\,dl\,ds$$
$$-\int_{S_3}\int_{\Gamma_3}[\lambda\nabla T\mathbf{n}+\alpha(T-T_f)]\delta T^*\,dl\,ds = 0 \quad (12)$$

where the functions $C(x,y,z,\tau)$, $\lambda(x,y,z,\tau)$, $w_1(x,y,z,\tau)$, $\alpha(x,y,z,\tau)$ are defined as symmetrized at the level $x = X/2$, $\tau = t/2$ according to the rule

$$\bar{f}(x,y,z,\tau) = \begin{cases} f(x,y,z,\tau), & 0<\tau<t/2, 0<x<X/2 \\ f(X-x,y,z,t-\tau), & t/2<\tau<t, X/2<x<X. \end{cases}$$

The latter constraint does not in the least restrict the possibilities of the resulting variational description, since one may take any values of the arguments $x = X/2$ and $\tau = t/2$, including those at which the solution of the problem with the real functions $C$, $Cw_1$, $\lambda$, $\alpha$ is of interest.

## 3. DERIVATION OF THE SET OF EULER EQUATIONS TO OBTAIN THE *n*TH APPROXIMATION TO THE TEMPERATURE FIELD IN THE FLOW

Let the $n$th approximation to the solution of problem (1)–(4) be presented in the form

$$T(x,y,z,\tau) = \sum_{i=1}^{n}\chi_i(x,y,z,\tau)\psi_i(x,\tau) \quad (13)$$

where $\chi_i(x,y,z,\tau)$ is a complete system of coordinate functions in the integrated region $\Omega$ selected to satisfy conditions (4) and $\psi_i(x,\tau)$ are the unknown functions $x$ and $\tau$. Note that the dependence $\chi_i(x,y,z,\tau)$ on $x,\tau$ results from the presence of the functions $\lambda(x,y,z,\tau)$, $T_w(x,y,z,\tau)$, $T_f(x,y,z,\tau)$, $q(x,y,z,\tau)$, $\alpha(x,y,z,\tau)$ in the boundary conditions.

The substitution of equation (13) into equation (12) gives

$$\delta J(T) = \int_v\Big\{\sum_{i=1}^{n}\psi_i(x,\tau)[(\lambda\chi'_{i,y})_y+(\lambda\chi'_{i,z})_z$$
$$-C\chi'_{i,\tau}-Cw_1\chi'_{i,x}-Cw_2\chi'_{i,y}-Cw_3\chi'_{i,z}]+q_v$$
$$-\sum_{i=1}^{n}\psi'_{i,x}Cw_1\chi_i-\sum_{i=1}^{n}\psi'_{i,\tau}C\chi_i\Big\}$$
$$\times\sum_{j=1}^{n}\delta\psi_j^*\chi_j\,dv-\int_{S_1}C(x,y,z,0)$$
$$\times\left[\sum_{i=1}^{n}\psi_i(x,0)\chi_i-T_0\right]\sum_{j=1}^{n}\delta\psi_j(X-x,t)\chi_j\,ds$$
$$-\int_{S_2}Cw_1(0,y,z,\tau)\left[\sum_{i=1}^{n}\psi_i(0,\tau)\chi_i-T_{en}\right]$$
$$\times\sum_{j=1}^{n}\delta\psi_j(X,t-\tau)\chi_j\,ds+\int_{S_3}\int_{\Gamma_1}\lambda$$
$$\times\left[\sum_{i=1}^{n}\psi_i(x,\tau)\chi_i-T_w\right]\sum_{j=1}^{n}\delta\psi_j^*\nabla\chi_j\mathbf{n}\,dl\,ds$$
$$-\int_{S_3}\int_{\Gamma_2}\left[\lambda\sum_{i=1}^{n}\psi_i(x,\tau)\nabla\chi_i\mathbf{n}+q\right]\sum_{j=1}^{n}\delta\psi_j^*\chi_j\,dl\,ds$$
$$-\int_{S_3}\int_{\Gamma_3}\Big\{\lambda\sum_{i=1}^{n}\psi_i(x,\tau)\nabla\chi_i\mathbf{n}+\alpha$$
$$\times\left[\sum_{i=1}^{n}\psi_i(x,\tau)\chi_i-T_f\right]\Big\}\sum_{j=1}^{n}\delta\psi_j^*\chi_j\,dl\,ds = 0. \quad (14)$$

Taking into account the independence of the variations $\delta\psi_j^*$, $\delta\psi_j(X-x,t)$, $\delta\psi_j(X,t-\tau)$ at any $j = 1,\ldots,n$, it is possible to obtain that the unknown functions $\psi_i(x,\tau)$ are the solution of the following Cauchy problem for the set of first-order partial differential (Euler) equations:

$$\sum_{i=1}^{n}[\psi_i(x,\tau)\cdot P_{ji}(x,\tau)+\psi'_{i,\tau}(x,\tau)\gamma_{ji}(x,\tau)+\psi'_{i,x}(x,\tau)\beta_{ji}(x,\tau)]+Q_j(x,\tau) = 0$$
$$\sum_{i=1}^{n}\psi_i(x,0)\delta_{ji}(x)+\phi_j(x) = 0$$
$$\sum_{i=1}^{n}\psi_i(0,\tau)d_{ji}(\tau)+a_j(\tau) = 0, j = 1,\ldots,n \quad (15)$$

where

$$P_{ji}(x,\tau) = \int_\Omega[(\lambda\chi'_{i,y})_y+(\lambda\chi'_{i,z})_z-C\chi'_{i,\tau}-Cw_1\chi'_{i,x}-Cw_2\chi'_{i,y}-Cw_3\chi'_{i,z}]\chi_j\,d\Omega+\int_{\Gamma_1}\lambda\chi_i\chi_j\,dl$$
$$-\int_{\Gamma_2}\lambda\nabla\chi_i\mathbf{n}\chi_j\,dl-\int_{\Gamma_3}(\lambda\nabla\chi_i\mathbf{n}+\alpha\chi_i)\chi_j\,dl$$

$$\gamma_{ji}(x,\tau) = -\int_\Omega C\chi_i\chi_j\,d\Omega$$

$$\beta_{ji}(x,\tau) = -\int_\Omega Cw_1\chi_i\chi_j\,d\Omega$$

$$Q_j(x,\tau) = \int_\Omega q_v\chi_j\,d\Omega-\int_{\Gamma_1}\lambda T_w\chi_j\,dl-\int_{\Gamma_2}q\chi_j\,dl+\int_{\Gamma_3}\alpha T_f\chi_j\,dl$$

$$\delta_{ji}(x) = -\int_\Omega C(x,y,z,0)\chi_i\chi_j\,d\Omega$$

$$\phi_j(x) = \int_\Omega C(x,y,z,0)T_0\chi_j\,d\Omega$$

$$d_{ji}(\tau) = -\int_\Omega Cw_1(0,y,z,\tau)\chi_i\chi_j\,d\Omega$$

$$a_j(\tau) = \int_\Omega C(0,y,z,\tau)T_{en}\chi_j\,d\Omega. \quad (16)$$

## 4. UNSTEADY-STATE HEAT TRANSFER IN A PLANE CHANNEL. AN EXAMPLE TO ILLUSTRATE A PRACTICAL APPLICATION OF THE THEORY

An example will now be given showing the construction of the set of Euler equations (15) and its solution for the case of unsteady-state heat transfer of a laminar flow of a Newtonian fluid in a plane channel when the rate of external heat transfer on both surfaces varies longitudinally in such a way that the corresponding Biot number is equal to

$$Bi = 2\tilde{x}/(1+\tilde{x})$$

and the external medium temperature varies periodically in time according to the law

$$T_f = T_0(1-\sin\omega Fo).$$

Here, the dimensionless variables $\tilde{x} = x/(Pe\, l_0)$, $Pe = w_0 l_0/a$, and $Fo = a\tau/l_0^2$ have been used which denote respectively the dimensionless longitudinal coordinate, Peclet and Fourier numbers based on the channel half-thickness $l_0$, mean flow rate $w_0$ and the thermal diffusivity of the medium $a = \lambda/C$.

Then, provided the temperatures at the inlet to the channel ($\tilde{x} = 0$) and the initial instant of time ($Fo = 0$) coincide, there is the following boundary-value problem:

$$\frac{\partial T(\xi,\tilde{x},Fo)}{\partial Fo} + (1-\xi^2)\frac{\partial T(\xi,\tilde{x},Fo)}{\partial \tilde{x}} = \frac{\partial^2 T(\xi,\tilde{x},Fo)}{\partial \xi^2}, \quad \tilde{x} > 0, 0 < \xi < 1, Fo > 0 \quad (17)$$

$$T(\xi,\tilde{x},0) = T_0, \quad 0 < \xi < 1, \tilde{x} > 0 \quad (18)$$

$$T(\xi,0,Fo) = T_0, \quad 0 < \xi < 1, Fo > 0 \quad (19)$$

$$-\left.\frac{\partial T(\xi,\tilde{x},Fo)}{\partial \xi}\right|_{\xi=1} = Bi[T(1,\tilde{x},Fo)-T_f], \quad \tilde{x} > 0, Fo > 0 \quad (20)$$

$$\left.\frac{\partial T(\xi,\tilde{x},Fo)}{\partial \xi}\right|_{\xi=0} = 0, \quad \tilde{x} > 0, Fo > 0 \quad (21)$$

where $y$ and $\xi = y/l_0$ are the dimensional and dimensionless transverse coordinate reckoned from the symmetry $y = 0(\xi = 0)$.

The substitution

$$T(\xi,\tilde{x},Fo) = u(\xi,\tilde{x},Fo) + T_0(1+\xi^2\tilde{x}\sin\omega Fo) \quad (22)$$

will reduce problem (17)–(21) to the homogeneous boundary conditions

$$\frac{\partial u}{\partial Fo} + (1-\xi^2)\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} + Q_v(\xi,\tilde{x},Fo) \quad (17')$$

$$u(\xi,\tilde{x},0) = 0 \quad (18')$$

$$u(\xi,0,Fo) = 0 \quad (19')$$

$$-\left.\frac{\partial u}{\partial \xi}\right|_{\xi=1} = Bi\,u\Big|_{\xi=1} \quad (20')$$

$$\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} = 0 \quad (21')$$

where

$$Q_v(\xi,\tilde{x},Fo) = T_0[\omega\xi^2\tilde{x}\cos\omega Fo + (1-\xi^2)\xi^2\sin\omega Fo - 2\tilde{x}\sin\omega Fo].$$

Problem (1)–(4) passes over into the particular case (17′)–(21′), if it is assumed that $\tau = Fo$, $\tilde{x} = x$, $y = \xi$, $C = 1$, $\lambda = 1$, $w_1 = 1-\xi^2$, $w_2 = w_3 = 0$, $T_z = 0$, $q_v = Q_v$, $T(x,y,z,0) = T(0,y,z,\tau) = 0$, $T_w(x,y,z,\tau) = 0$, $q(x,y,z,\tau) = 0$, $\alpha(x,y,z,\tau) = 2\tilde{x}/(1+\tilde{x})$, $T_f(x,y,z,\tau) = 0$, $T(x,y,z,\tau) = u(\xi,\tilde{x},Fo)$.

In formula (22) the unknown function $u(\xi,\tilde{x},Fo)$ will be represented in the form of equation (13)

$$u(\xi,\tilde{x},Fo) = \sum_{i=1}^{n} \chi_i(\xi,\tilde{x},Fo)\psi_i(\tilde{x},Fo).$$

Then, on the basis of equation (12), the following coefficients and free numbers will be obtained in the set of Euler equations (15) for the problem at hand

$$P_{ji} = \int_0^1 \chi''_{i,\xi\xi}\chi_j\,d\xi - \int_0^1 (1-\xi^2)\chi_i\chi_j\,d\xi$$

$$\gamma_{ji} = \delta_{ji} = -\int_0^1 \chi_i\chi_j\,d\xi$$

$$\beta_{ji} = d_{ji} = -\int_0^1 (1-\xi^2)\chi_i\chi_j\,d\xi$$

$$Q_j = \int_0^1 Q_v(\xi,\tilde{x},Fo)\chi_j\,d\xi$$

$$\phi_j = a_j = 0. \quad (23)$$

In the above formulae the coordinate functions $\chi_i$, $\chi_j$ are selected in the following form to satisfy boundary conditions (20′) and (21′):

$$\chi_i = a_i\xi^{2(i-1)} + b_i\xi^{2i}, \quad i = 1,\ldots,n$$

where $a_i = 1$, $b_i = -(2i-2+Bi)/(2i+Bi)$.

Then, the functions in equations (23) will acquire the following form:

$$P_{ji} = \frac{(2i-2)(2i-3)}{2i+2j-5} + \frac{2ib_i(2i-1)}{2i+2j-3} + \frac{b_j(2i-2)(2i-3)}{2i+2j-3} + \frac{2ib_i(2i-1)}{2i+2j-1} + \frac{1}{(1+i\tilde{x}+\tilde{x})^2}\left(\frac{1}{2i+2j-1} + \frac{b_i}{2i+2j+1} - \frac{1}{2i+2j+1} - \frac{b_j}{2i+2j+3}\right)$$

$$\gamma_{ji} = \delta_{ji} = -\left(\frac{1}{2i+2j-3} + \frac{b_i+b_j}{2i+2j-1} + \frac{b_ib_j}{2i+2j+1}\right)$$

$$\beta_{ji} = d_{ji} = -\gamma_{ji} + \frac{1}{2i+2j-1} + \frac{b_i+b_j}{2i+2j+1} + \frac{b_ib_j}{2i+2j+3}$$

Table 1. Comparison of approximate analytical and grid values of $T(\xi, \tilde{x}, Fo)$

| $\tilde{x}$ | Number of approximations 1st | 2nd | 5th | 10th | Grid | Number of approximations 1st | 2nd | 5th | 10th | Grid | $Fo$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\xi = 0.1$ | | | | | $\xi = 0.5$ | | | | |
| 0.1 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.0000 | 1.000 | |
| 0.3 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.0005 | 1.0008 | 1.001 | 1.001 | 1.001 | |
| 0.5 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.0005 | 1.0018 | 1.002 | 1.002 | 1.002 | 0.10 |
| 0.7 | 1.000 | 1.0005 | 1.001 | 1.001 | 1.001 | 1.001 | 1.0036 | 1.004 | 1.004 | 1.004 | |
| 0.9 | 1.000 | 1.0006 | 1.001 | 1.001 | 1.001 | 1.003 | 1.0048 | 1.005 | 1.006 | 1.005 | |
| 1.0 | 1.000 | 1.0006 | 1.001 | 1.001 | 1.001 | 1.003 | 1.005 | 1.005 | 1.006 | 1.005 | |
| | | $\xi = 0.3$ | | | | | $\xi = 0.6$ | | | | |
| 0.1 | 1.000 | 1.000 | 1.000 | 1.00 | 1.000 | 1.0009 | 1.000 | 1.000 | 1.001 | 1.001 | |
| 0.3 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.001 | 1.002 | 1.002 | 1.002 | 1.002 | |
| 0.5 | 1.000 | 1.000 | 1.001 | 1.001 | 1.001 | 1.003 | 1.0036 | 1.004 | 1.004 | 1.004 | 0.10 |
| 0.7 | 1.0008 | 1.001 | 1.001 | 1.001 | 1.001 | 1.004 | 1.006 | 1.006 | 1.006 | 1.006 | |
| 0.9 | 0.0017 | 1.002 | 1.002 | 1.002 | 1.002 | 1.005 | 1.007 | 1.008 | 1.008 | 1.008 | |
| 1.0 | 1.0018 | 1.002 | 1.002 | 1.002 | 1.002 | 1.005 | 1.009 | 1.009 | 1.010 | 1.009 | |
| | | $\xi = 0.7$ | | | | | $\xi = 0.9$ | | | | |
| 0.1 | 1.0006 | 1.0008 | 1.001 | 1.001 | 1.001 | 1.0012 | 1.002 | 1.003 | 1.003 | 1.003 | |
| 0.3 | 1.0027 | 1.0035 | 1.004 | 1.004 | 1.004 | 1.007 | 1.009 | 1.010 | 1.010 | 1.010 | |
| 0.5 | 1.0052 | 1.0063 | 1.007 | 1.007 | 1.007 | 1.014 | 1.016 | 1.017 | 1.018 | 1.017 | |
| 0.7 | 1.008 | 1.0097 | 1.010 | 1.010 | 1.010 | 1.022 | 1.024 | 1.025 | 1.026 | 1.025 | |
| 1.0 | 1.0127 | 1.0146 | 1.015 | 1.016 | 1.015 | 1.032 | 1.036 | 1.036 | 1.037 | 1.036 | |
| | | $\xi = 0.8$ | | | | | $\xi = 1.0$ | | | | 0.10 |
| 0.1 | 1.0017 | 1.002 | 1.002 | 1.002 | 1.002 | 1.0037 | 1.0043 | 1.005 | 1.005 | 1.005 | |
| 0.3 | 1.0054 | 1.006 | 1.006 | 1.006 | 1.006 | 1.013 | 1.0145 | 1.015 | 1.015 | 1.015 | |
| 0.5 | 1.013 | 1.0014 | 1.014 | 1.014 | 1.011 | 1.025 | 1.0256 | 1.026 | 1.026 | 1.026 | |
| 0.7 | 1.015 | 1.016 | 1.016 | 1.016 | 1.016 | 1.036 | 1.0374 | 1.038 | 1.038 | 1.038 | |
| 0.9 | 1.021 | 1.021 | 1.021 | 1.021 | 1.021 | 1.047 | 1.0488 | 1.049 | 1.049 | 1.049 | |
| 1.0 | 1.023 | 1.023 | 1.023 | 1.025 | 1.023 | 1.054 | 1.055 | 1.055 | 1.055 | 1.055 | |
| | | $\xi = 0.1$ | | | | | $\xi = 0.5$ | | | | |
| 0.1 | 1.0061 | 1.0073 | 1.008 | 1.008 | 1.008 | 1.0193 | 1.0206 | 1.021 | 1.022 | 1.021 | |
| 0.3 | 1.0053 | 1.0056 | 1.057 | 1.0058 | 1.058 | 1.097 | 1.102 | 1.104 | 1.105 | 1.106 | |
| 0.5 | 1.118 | 1.130 | 1.137 | 1.139 | 1.138 | 1.206 | 1.212 | 1.219 | 1.219 | 1.219 | |
| 0.7 | 1.222 | 1.228 | 1.235 | 1.234 | 1.233 | 1.295 | 1.337 | 1.345 | 1.346 | 1.343 | |
| 0.9 | 1.316 | 1.328 | 1.330 | 1.332 | 1.332 | 1.434 | 1.455 | 1.466 | 1.467 | 1.469 | |
| 1.0 | 1.368 | 1.370 | 1.382 | 1.381 | 1.382 | 1.516 | 1.529 | 1.534 | 1.532 | 1.532 | |
| | | $\xi = 0.3$ | | | | | $\xi = 0.6$ | | | | 0.90 |
| 0.1 | 1.0084 | 1.011 | 1.013 | 1.012 | 1.012 | 1.022 | 1.026 | 1.029 | 1.028 | 1.028 | |
| 0.3 | 1.066 | 1.0707 | 1.071 | 1.072 | 1.073 | 1.120 | 1.128 | 1.127 | 1.129 | 1.129 | |
| 0.5 | 1.151 | 1.160 | 1.165 | 1.166 | 1.165 | 1.238 | 1.256 | 1.256 | 1.257 | 1.258 | |
| 0.7 | 1.247 | 1.259 | 1.268 | 1.268 | 1.269 | 1.375 | 1.386 | 1.394 | 1.398 | 1.295 | |
| 0.9 | 1.349 | 1.370 | 1.375 | 1.376 | 1.377 | 1.511 | 1.531 | 1.531 | 1.533 | 1.533 | |
| 1.0 | 1.407 | 1.422 | 1.432 | 1.430 | 1.431 | 1.588 | 1.600 | 1.602 | 1.604 | 1.603 | |
| | | $\xi = 0.7$ | | | | | $\xi = 0.9$ | | | | |
| 0.1 | 1.029 | 1.036 | 1.036 | 1.038 | 1.037 | 1.052 | 1.061 | 1.060 | 1.062 | 1.061 | |
| 0.3 | 1.148 | 1.157 | 1.160 | 1.162 | 1.158 | 1.220 | 1.229 | 1.232 | 1.230 | 1.231 | |
| 0.5 | 1.295 | 1.301 | 1.308 | 1.305 | 1.304 | 1.407 | 1.420 | 1.423 | 1.422 | 1.420 | |
| 0.7 | 1.431 | 1.452 | 1.453 | 1.458 | 1.457 | 1.596 | 1.609 | 1.610 | 1.613 | 1.612 | |
| 0.9 | 1.585 | 1.602 | 1.606 | 1.609 | 1.611 | 1.783 | 1.795 | 1.806 | 1.804 | 1.803 | |
| 1.0 | 1.658 | 1.682 | 1.682 | 1.684 | 1.687 | 1.842 | 1.888 | 1.901 | 1.900 | 1.898 | |
| | | $\xi = 0.8$ | | | | | $\xi = 1.0$ | | | | 0.90 |
| 0.1 | 1.041 | 1.047 | 1.047 | 1.049 | 1.048 | 1.068 | 1.075 | 1.077 | 1.078 | 1.077 | |
| 0.3 | 1.179 | 1.191 | 1.192 | 1.194 | 1.192 | 1.255 | 1.269 | 1.274 | 1.275 | 1.275 | |
| 0.5 | 1.345 | 1.357 | 1.358 | 1.359 | 1.358 | 1.469 | 1.485 | 1.490 | 1.492 | 1.489 | |
| 0.7 | 1.493 | 1.528 | 1.532 | 1.529 | 1.529 | 1.688 | 1.698 | 1.702 | 1.705 | 1.704 | |
| 0.9 | 1.674 | 1.696 | 1.700 | 1.702 | 1.700 | 1.898 | 1.909 | 1.916 | 1.919 | 1.918 | |
| 1.0 | 1.770 | 1.775 | 1.780 | 1.782 | 1.785 | 2.013 | 2.022 | 2.024 | 2.024 | 2.023 | |

$$Q_j = -T_0\left[\omega\tilde{x}\cos\omega Fo\left(\frac{1}{2j+1}+\frac{b_j}{2j+3}\right)\right.$$
$$+\sin\omega Fo\left(\frac{1}{2j+1}+\frac{b_j-1}{2j+3}-\frac{b_j}{2j+5}\right)$$
$$\left.-2\tilde{x}\sin\omega Fo\left(\frac{1}{2j-1}+\frac{b_j}{2j+1}\right)\right]. \quad (24)$$

The numerical solution of the set of equations (15), written for the unknown $\psi_i$ in matrix form

$$\Gamma\frac{\partial\psi}{\partial Fo}+B\frac{\partial\psi}{\partial x}=P\psi+\mathbf{Q} \quad (25)$$

was found with the aid of the difference scheme predictor–corrector [6], which approximates with second-order accuracy with respect to both variables $\tilde{x}$ and $Fo$. In this case, the term of the form $P\psi$ was approximated nonexplicitly, whereas $B(\partial\psi/\partial\tilde{x})$ was approximated explicitly.

The results of computer calculations for various approximations to the solution of problem (17)–(21) give practical coincidence with the values of $T(\xi,\tilde{x},Fo)$ obtained from the solution of the initial boundary-value problem with the analog-to-digital conversion over all the variables, beginning with $Fo \geqslant 0.02$ and $\tilde{x} \geqslant 0.02$ already for the first approximation (the discrepancy does not exceed 5%). The subsequent approximations improve these results substantially. For practical applications, it is important to emphasize the convergence of approximations even in the case of periodic variation in time of the external effect: as the number of the approximation increases, their asymptotic approach to 'exact' values is observed (Table 1).

It should be noted that the numerical solution by the method considered involves, nevertheless, an analytical part in the form of the coordinate functions $\chi_i$ and moreover, due to the reduction of the order of derivatives in the equation of the process and of the analytic character in the system of equations (15), some economy of machine time is effected, approximately by one or two orders of magnitude.

It should be emphasized that the proposed numerical method for determining temperature fields in the flow, which is based on the use of a variational description, is also applicable in the case when the velocity vector components are not preassigned in the form of the known functions of coordinates and time, but can be found by solving the corresponding boundary-value problem with the use of variable directions on each time half-layer, on the first half-layer a hydrodynamic problem is solved, and on the second half-layer a thermal problem which requires less machine time.

## 5. CONCLUSION

The variational description of the process of unsteady-state convective heat transfer in a channel of complex cross-section, with the thermophysical characteristics of the medium, velocity vector components, volumetric heat generation sources and the parameters of boundary conditions being dependent on the arguments, can be successfully applied to determine temperature fields in the internal flows.

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## APPENDIX. CALCULATION OF THE FIRST VARIATION OF THE FUNCTIONAL WHEN CONSTRUCTING A VARIATIONAL DESCRIPTION OF UNSTEADY-STATE HEAT TRANSFER IN NON-STABILIZED FLOW IN CHANNELS OF COMPLEX CROSS-SECTION

Calculate the first variation $J_1(T)$, defined by formula (6), to obtain

$$\delta J_1=\delta J_{1,1}+\delta J_{1,2}=\int_v\left[R\delta T_\tau+G\delta T_x+B\delta T+D\delta T_y+E\delta T_z\right.$$
$$\left.+\frac{\partial}{\partial z}(A\delta T_z)+\frac{\partial}{\partial y}(A\delta T_y)\right]T^*\,dv+\int_v\left[RT_\tau+GT_x+BT\right.$$
$$\left.+DT_y+ET_z+\frac{\partial}{\partial z}(AT_z)+\frac{\partial}{\partial y}(AT_y)+2Q\right]\delta T^*\,dv. \quad (A1)$$

Applying the integration by parts to the first two terms $\delta J_{1,1}$ and the Ostrogradsky–Green theorem to the last terms, and using the property of the convolution commutativity the quantity $\delta J_{1,1}$ can be obtained in the form

$$\delta J_{1,1}=\int_{S_1}R\delta T\cdot T^*\Big|_0^t\,ds-\int_v(RT^*)'_\tau\delta T\,dv$$
$$+\int_{S_2}GT^*\delta T\Big|_0^X\,ds-\int_v(GT^*)'_x\delta T\,dv+\int_v BT^*\delta T\,dv$$
$$+\int_{S_3}\int_\Gamma(Dn_1\delta T+En_2\delta T+An_1\delta T_y+An_2\delta T_z)T^*\,dl\,ds$$
$$-\int_v\{[(DT^*)'_y+(ET^*)'_z]\delta T+AT^*_y\delta T_y+AT^*_z\delta T_z\}\,dv$$
$$=\int_{S_1}RT^*\delta T\Big|_0^t\,ds+\int_{S_2}G\delta T\cdot T^*\Big|_0^X\,ds+\int_{S_3}\int_\Gamma\{[(Dn_1$$
$$+En_2)\delta T+A\nabla\delta T\cdot\mathbf{n}]T^*-AT^*_yn_1\delta T-AT^*_zn_2\delta T\}\,dl\,ds$$
$$+\int_v\left[\frac{\partial}{\partial y}(AT^*_y)+\frac{\partial}{\partial z}(AT^*_z)-(DT^*)'_y-(ET^*)'_z+BT^*\right.$$
$$\left.-(GT^*)'_x-(RT^*)'_\tau\right]\delta T\,dv. \quad (A2)$$

The use of formulae (A1) and (A2) yields

$$\delta J_1 = \int_v \Big\{ \frac{\partial}{\partial y}[(A+A^*)T_y] + \frac{\partial}{\partial z}[(A+A^*)T_z] + [B-D_y^*-E_z^*+B^*-(B_x)^*-(R_\tau)^*]T+(G+G^*)T_x + (D-D^*)T_y+(E-E^*)T_z+(R+R^*)T_\tau+2Q \Big\}\delta T^*\,dv + \int_{S_1} R\delta T\cdot T^*\Big|_0^t ds + \int_{S_2} G\delta T\cdot T^*\Big|_0^X ds + \int_{S_3}\int_\Gamma \{[(Dn_1 + En_2)\delta T + A\nabla\delta T\mathbf{n}]T^* - A\delta T\cdot T^*\}\,dl\,ds. \quad (A1')$$

The first variation $J_2$ from equation (7) is thus

$$\delta J_2 = \int_{S_2} [a\delta T(X-x,t)T(x,0) - aT(X-x,t)\delta T(x,0) + b\delta T(X-x,t)]\,ds. \quad (A3)$$

Supplementing equation (A3) with the term from equation (A1′), having first made the substitution of limits and then using the property of the convolution commutativity

$$j_2 = \int_{S_1} R\delta T\cdot T^*\Big|_0^t ds = \int_{S_1} [R(x,t)\delta T(x,t)T(X-x,0) - R(x,0)\delta T(x,0)T(X-x,t)]\,ds = \int_{S_1} [R(X-x,t)\delta T \times (X-x,t) - R(X-x,0)\delta T(X-x,0)T(x,t)]\,ds$$

to obtain

$$\delta J_2 + j_2 = \int_{S_1} \{\{[R(X-x,t)+a]T(x,0)+b\}\delta T(X-x,t) + [a-R(x,0)]T(X-x,t)\delta T(x,0)\}\,ds. \quad (A4)$$

Letting in equation (A4)

$$a = R(x,0), \quad b = -[R(X-x,t)+a]T_0$$

gives finally

$$\delta J_2 + j_2 = \int_{S_1} [R(x,0)+R(X-x,t)] \times [T(x,0)-T_0]\delta T(x,0)\,ds. \quad (A4')$$

The first variation $J_3$ from equation (8) is thus

$$\delta J_3 = \int_{S_2} [\tilde{a}\delta T(X,t-\tau)T(0,\tau) + \tilde{a}T(X,t-\tau)\delta T(0,\tau) + \tilde{b}\delta T(X,t-\tau)]\,ds. \quad (A5)$$

Supplementing $\delta J_3$ with the term from equation (A1′), having first made in it the substitution of limits and then using the property of the convolution commutativity

$$j_3 = \int_{S_2} G\delta T\cdot T^*\Big|_0^X ds = \int_{S_2} [G(X,\tau)\delta T(X,\tau)T(0,t-\tau) - G(0,\tau)\delta T(0,\tau)T(X,t-\tau)]\,ds = \int_{S_2} [G(X,t-\tau)\delta T \times (X,t-\tau)T(0,\tau) - G(0,t-\tau)\delta T(0,t-\tau)T(X,\tau)]\,ds$$

to obtain

$$\delta J_3 + j_3 = \int_{S_2} \{[\tilde{a}T(0,\tau)+\tilde{b}+G(X,t-\tau)T(0,\tau)]\delta T(X,t-\tau) + [\tilde{a}T(X,t-\tau) - G(0,\tau)T(X,t-\tau)]\delta T(0,\tau)\}\,ds. \quad (A6)$$

Assuming in equation (A6) that

$$\tilde{a} = G(0,\tau), \quad \tilde{b} = -[a+G(X,t-\tau)]T_{en}$$

finally yields

$$\delta J_3 + j_3 = \int_{S_2} [G(0,\tau)+G(X,t-\tau)][T(0,\tau)-T_{en}]\delta T(0,\tau). \quad (A6')$$

The term

$$\int_{S_3}\int_\Gamma dl\,ds$$

in formulae (A2) can be represented as

$$\int_{S_3}\int_\Gamma dl\,ds = \int_{S_3}\int_{\Gamma_1} dl\,ds + \int_{S_3}\int_{\Gamma_2} dl\,ds + \int_{S_3}\int_{\Gamma_3} dl\,ds. \quad (A7)$$

The variation $J_4$ from equation (9) added to the first term on the right-hand side of equation (A7) is as follows:

$$\delta J_4 + \int_{S_3}\int_{\Gamma_1} dl\,ds = \int_{S_3}\int_{\Gamma_1} \{[(Dn_1+En_2)\delta T + A(\delta T_y n_1 + \delta T_z n_2)]T^* - (T_y^* n_1 + T_z^* n_2)A\delta T + p_1(T-T_w)\delta\nabla T^*\mathbf{n} + (p_1+p_2)(T_y^* n_1 + T_z^* n_2)\delta T + p_3(T^*\delta T + T\cdot\delta T^*) + p_2 T(\delta T_y^* n_1 + \delta T_z^* n_2)\}\,dl\,ds. \quad (A8)$$

The sum of variations $J_5$ from equation (10) with the second term on the right-hand side of equation (A7) gives

$$\delta J_5 + \int_{S_3}\int_{\Gamma_2} dl\,ds = \int_{S_3}\int_{\Gamma_3} \{[(Dn_1+En_2)\delta T + (\delta T_y n_1 + \delta T_z n_2)A]T^* - A(T_y^* n_1 + T_z^* n_2)\delta T + a_1(\lambda\nabla T\mathbf{n}+q)\delta T^* + a_1\lambda T_y n_1 T^* + a_1\lambda\delta T_z n_2 T^* + a_2(T_y^* n_1 + T_z^* n_2)\delta T + a_3(T\delta T^* + T^*\delta T) + a_2(\delta T_y^* n_1 + \delta T_z^* n_2)T\}\,dl\,ds. \quad (A9)$$

In the same way the sum of variation of $J_6$ from equation (11) and of the third term on the right-hand side of equation (A7) yields

$$\delta J_6 + \int_{S_3}\int_{\Gamma_3} dl\,ds = \int_{S_3}\int_{\Gamma_3} \{b_1[\lambda\nabla T\mathbf{n} + \alpha(T-T_f)]\delta T^* + b_1\lambda T^*(\delta T_y n_1 + \delta T_z n_2) + b_1\alpha T^*\delta T + g_1\delta T^* T_y n_1 + g_1 n_1(\delta T^* T_z + T^*\delta T_z) + g_1 n_1 \delta T_y T^* + g_1 T^*\delta T_z n_2 + g_2(T^*\delta T + T\delta T^*) + [(Dn_1+En_2)\delta T + A\delta T_y n_1 + A\delta T_z n_2]T^* - A\delta T(T_y^* n_1 + T_z^* n_2)\}\,dl\,ds. \quad (A10)$$

Thus, all the above transformations give the first variation of functional (5) in the form

$$\delta J(T) = \int_v \Big\{(R+R^*)T_\tau + (G+G^*)T_x + (D-D^*)T_y + (E-E^*)T_z + \frac{\partial}{\partial y}[(A+A^*)T_y] + \frac{\partial}{\partial z}[(A+A^*)T_z] + 2Q + [B-D_y^*-E_z^*+B^*-(B_x)^*-(R_\tau)^*]T\Big\}\delta T^*\,dv + \int_{S_1} [R(X-x,t)+R(x,0)][T(x,0)-T_0]\delta T(x,0)\,ds + \int_{S_2} [G(0,\tau)+G(X,t-\tau)][T(0,\tau)-T_{en}]\delta T(0,\tau)\,ds + \int_{S_3}\int_{\Gamma_1} \{[(Dn_1+En_2)\delta T + A\delta\nabla T\mathbf{n}]T^* - A\delta T\nabla T^*\mathbf{n} + p_1(T-T_w)\cdot\delta\nabla T^*\mathbf{n} + (p_1+p_2)\delta T\nabla T^*\mathbf{n} + p_2 T(\tau)\delta\nabla T^*\mathbf{n} + p_3(T^*\delta T + T\delta T^*)\}\,dl\,ds + \int_{S_3}\int_{\Gamma_2} \{[(Dn_1+En_2)\delta T$$

$$+A\delta\nabla T\mathbf{n}]T^*-A\delta T\nabla T^*\mathbf{n}+a_1(\lambda\nabla T\mathbf{n}+q)\delta T^*$$
$$+a_1\lambda T^*\delta\nabla T\mathbf{n}+a_2\delta T\nabla T^*\mathbf{n}+a_3(T\delta T^*+T^*\delta T)$$
$$+a_2T\delta\nabla T^*\mathbf{n}\}\,\mathrm{d}l\,\mathrm{d}s+\int_{S_3}\int_{\Gamma_3}\{[(Dn_1+En_2)\delta T$$
$$+A\delta\nabla T\mathbf{n}]T^*-A\delta T\nabla T^*\mathbf{n}+b_1[\lambda\nabla T\mathbf{n}+\alpha(T-T_f)]\delta T^*$$
$$+b_1\lambda T^*\delta\nabla T\mathbf{n}+b_1\alpha T^*\delta T+g_1\delta T^*\nabla T\mathbf{n}+g_1T^*\delta\nabla T\cdot\mathbf{n}$$
$$+g_2(T\delta T^*+T^*\delta T)\}\,\mathrm{d}l\,\mathrm{d}s,$$
$$n_1=\cos(n,y),\quad n_2=\cos(n,z). \qquad \text{(A11)}$$

In order that only the equations of problem (1)–(4) could convert $\delta J(T)$ to zero, taking into account the independency of the variations $\delta T$ and $\delta\nabla T\mathbf{n}$, it is necessary and sufficient that the following relations could hold:

$$R+R^*=-2C,\quad G+G^*=-2Cw_1,\quad D-D^*=-2Cw_2,$$
$$E-E^*=-2Cw_3,\quad A+A^*=2\lambda,$$
$$B-D_y^*-E_y^*+B^*-(G_x)^*-(R_\tau)^*=0,\quad Q=q_v,$$
$$Dn_1+En_2+p_3+p_3^*=0,\quad A+p_2=0,$$
$$-A+p_1+p_2^*=0,\quad Dn_1+En_2+a_3+a_3^*=0,$$
$$A+a_1\lambda+a_2^*=0,\quad -A+a_2^*=0,$$
$$Dn_1+En_2+g_2+b_1\alpha+g_2^*=0,\quad b_1\lambda+g_1+A=0,$$
$$g_1^*-A=0. \qquad \text{(A12)}$$

Assuming equations (A12) to be valid, obtain the first sought-after variation in the form

$$\delta J(T)=2\int_v\Big[-CT_\tau-Cw_1T_x-Cw_2T_y-Cw_3T_z+\frac{\partial}{\partial y}(\lambda T_y)$$
$$+\frac{\partial}{\partial z}(\lambda T_z)+q_v\Big]\delta T^*\,\mathrm{d}v-2\int_{S_1}C(x,0)[T(x,0)$$
$$-T_0]\delta T(x,0)\,\mathrm{d}s-2\int_{S_2}Cw_1(0,\tau)[T(0,\tau)-T_{en}]$$
$$\cdot\delta T(0,\tau)\,\mathrm{d}s+\int_{S_3}\int_{\Gamma_1}p_1(T-T_w)\delta\nabla T^*\mathbf{n}\,\mathrm{d}l\,\mathrm{d}s$$
$$+\int_{S_3}\int_{\Gamma_2}a_1(\lambda\nabla T\mathbf{n}+q)\delta T^*\,\mathrm{d}l\,\mathrm{d}s+\int_{S_3}\int_{\Gamma_3}b_1[\lambda\nabla T\mathbf{n}$$
$$+\alpha(T-T_f)]\delta T^*\,\mathrm{d}l\,\mathrm{d}s. \qquad \text{(A11')}$$

Based on equations (A12) in formula (A11) the following substitutions were made

$$R(x,0)+R(X-x,t)=-2C(x,0)$$
$$G(0,\tau)+G(X,t-\tau)=-2Cw_1(0,\tau)$$

since it is assumed that the following relations from equations (A12):

$$R+R^*=-2C,\quad G+G^*=-2Cw_1,\quad A+A^*=2\lambda$$

are realized on symmetrization of functions $C$, $Cw_1$, $\lambda$ at the level $x=X/2$, $\tau=t/2$

$$\bar{f}(x,y,z,\tau)=\begin{cases}f(x,y,z,\tau), & 0<\tau<t/2,\ 0<x<X/2\\ f(X-x,y,z,t-\tau), & t/2<\tau<t,\ X/2<x<X.\end{cases} \qquad \text{(A13)}$$

It is in this form (without the upper bar) that functions $C$, $Cw_1$, $\lambda$ are understood further.

The use of the rest relations from equations (A12) with allowance for the above described symmetrization of the function $\alpha(x,y,z,\tau)$ leads to the determination of the functions $p_1, a_1, b_1$: $p_1=2\lambda$, $a_1=b_1=-2$.

## SOLUTION VARIATIONNELLE DU PROBLEME DE CONVECTION THERMIQUE VARIABLE DANS UN CANAL

**Résumé**—A l'aide d'une fonctionnelle de type convolution, la description variationnelle est donnée pour le mécanisme de convection thermique variable dans des canaux de section droite complexe avec dépendance arbitraire vis-à-vis du temps et de l'espace, du vecteur vitesse, des propriétés thermophysiques du fluide, de la source volumétrique de chaleur et des paramètres des conditions aux limites. A partir de la méthode Galerkin–Kantorovich permettant la réduction en équations différentielles, on écrit un système correspondant des équations d'Euler dont la solution (analytique ou numérique) est nécessaire pour déterminer le champ de température dans chaque cas spécifique. On fournit un exemple pour illustrer une solution numérique du problème posé.

## VARIATIONSLÖSUNG DES PROBLEMS DES INSTATIONÄREN, KONVEKTIVEN WÄRMETRANSPORTS IN EINEM KANAL

**Zusammenfassung**—Durch die Anwendung eines Gleichungssystems wird die Beschreibung des instationären, konvektiven Wärmetransports in Kanälen mit komplizierten Querschnitten für unterschiedliche Fälle ermöglicht. Als Veränderliche treten der Geschwindigkeitsvektor, die thermophysikalischen Eigenschaften des Mediums, volumetrische Wärmeproduktion durch Quellen, sowie die Parameter der Randbedingungen in Raum und Zeit auf. Basierend auf der Galerkin–Kantorovich Methode, die eine Reduktion auf gewöhnliche Differentialgleichungen beinhaltet, wird ein System von korrespondierenden Euler-Gleichungen aufgestellt. Die Lösung dieses Systems (analytisch oder numerisch) ist notwendig um das Temperaturfeld in jedem speziellen Fall zu bestimmen. Um eine numerische Lösung des angeführten Problems zu veranschaulichen, wird ein Beispiel gegeben.

## ВАРИАЦИОННОЕ РЕШЕНИЕ ЗАДАЧИ НЕСТАЦИОНАРНОГО КОНВЕКТИВНОГО ТЕПЛООБМЕНА В КАНАЛЕ

**Аннотация**—С использованием функционала типа свертки построено вариационное описание процесса нестационарного конвективного теплообмена в каналах сложного поперечного сечения с произвольной зависимостью вектора скорости, теплофизических характеристик среды, источника объемного тепловыделения и параметров краевых условий от координат и времени. Основываясь на методе Галеркина–Канторовича приведения к обыкновенным дифференциальным уравнениям, выписана соответствующая система уравнений Эйлера, решение которой (численное или аналитическое) необходимо для определения температурного поля в каждом конкретном случае. Дан пример получения численного решения сформулированной задачи.